MATH 141 HOMEWORK 0 DUE DATE: NOT GRADED

Name: Solutions.

This homework assignment will not be graded. It is expected that you are comfortable with the material in this assignment. This covers the entirety of Chapter 1 in the textbook. If you have questions regarding any of this, feel free to ask during office hours. When writing solutions, present your answers clearly and neatly.

## Questions you should be able to answer

1. What is a function? What is its domain? Its range? Give examples.
2. What is the graph of a real-valued function of a real variable? What is the vertical line test?
3. What is a piecewise-defined function? Give examples.
4. What are the important types of functions frequently encountered in calculus? Give an example of each type.
5. What is meant by an increasing function? A decreasing function? Give an example of each.
6. What is an even function? An odd function? What symmetry properties do the graphs of such functions have? Give an example of a function that is neither odd nor even.
7. If $f$ and $g$ are real-valued functions, how are the domains of $f+g$, $f-g$, $fg$ and $f/g$ related to the domains of $f$ and $g$?
8. When is it possible to compose one function with another? Does the order in which functions are composed ever matter?
9. How do you change the equation $y = f(x)$ to shift its graph vertically up or down by $|k|$ units? Horizontally to the left or right?
10. How do you change the equation $y = f(x)$ to compress or stretch the graph by a factor $c > 1$? Reflect the graph across a coordinate axis?
11. Graph the six basic trigonometric functions. What symmetries do the graphs have?
12. What is an exponential function? What laws of exponents do they obey? How does it differ from a simple power function like $x^n$? What kind of real-world phenomena are modeled by exponential functions?
13. What functions have inverses? How do you know if two functions $f$ and $g$ are inverse of one another?
14. How are domains, ranges and graphs of functions and their inverses related?
15. What is a logarithmic function? What properties does it satisfy? What is the natural logarithm function? What does the graph look like?
16. How are the inverse trigonometric functions defined? How can you sometimes use right triangles to find values of these functions?

## Practice Problems

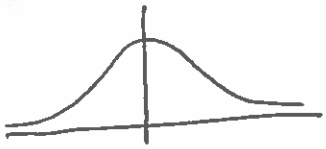
1. Determine if whether the graph of the function is symmetric about the $y$-axis, the origin, or neither.

   (a) $y = x^{2/5}$

   $y = (x^{1/5})^2$ ↑ square $\Rightarrow$ even.

   Answer: Symmetric in y-axis

   (b) $y = e^{-x^2}$

   Answer: Symmetric in y-axis

2. Determine whether the function is odd, even, or neither.

   (a) $y = x^5 - x^3 - x$

   $f(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -f(x)$

   Answer: Odd

   (b) $y = x - \sin(x)$ $\quad f(-x) = (-x) - \sin(-x)) = -x + \sin(x) = -f(x)$

   $x$ odd
   $\sin(x)$ odd

   Answer: Odd.

   (c) $y = x\cos(x)$

   $x$ odd
   $\cos(x)$ even

   $f(-x) = (-x)\cos(-x) = -x\cos(x) = -f(x)$

   Answer: Odd.

3. If $f(a - x) = f(a + x)$, show that $g(x) := f(x + a)$ is even.

   $g(-x) = f(-x+a) = f(x+a) = g(x)$.

   Answer:

4. Determine the domain and range of the function.

   (a) $y = -2 + \sqrt{1 - x}$

   shift down 2.

   $1 - x \geq 0 \Rightarrow 1 \geq x$

   Answer: D: $(-\infty, 1]$ R: $[-2, \infty)$

   (b) $y = \sqrt{16 - x^2}$

   $16 - x^2 \geq 0$

   $\Rightarrow -4 \leq x \leq 4$

   Answer: D: $[-4, 4]$ R: $[0, 4]$.

Cont.

(c) $y = \ln(x-3)+1$ ← shift up

$x - 3 > 0$

$\Rightarrow x > 3$

Answer: D: $(3, \infty)$, R $(1, \infty)$

5. Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{\sqrt{x+2}}$. Find

(a) $(f \circ g)(-1)$

$g(-1) = \frac{1}{\sqrt{-1+2}} = \frac{1}{\sqrt{1}} = 1$ $\quad f(g(-1)) = f(1) = \frac{1}{1} = 1$

Answer: 1

(b) $(g \circ f)(2)$

$f(2) = \frac{1}{2}$ $\quad g(f(2)) = g(\frac{1}{2}) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{\frac{5}{2}}} = \sqrt{\frac{2}{5}}$

Answer: $\sqrt{\frac{2}{5}}$

(c) $(f \circ f)(x)$

$f(f(x)) = f(\frac{1}{x}) = \frac{1}{\frac{1}{x}} = x$

Answer: $(f \circ f)(x) = x$

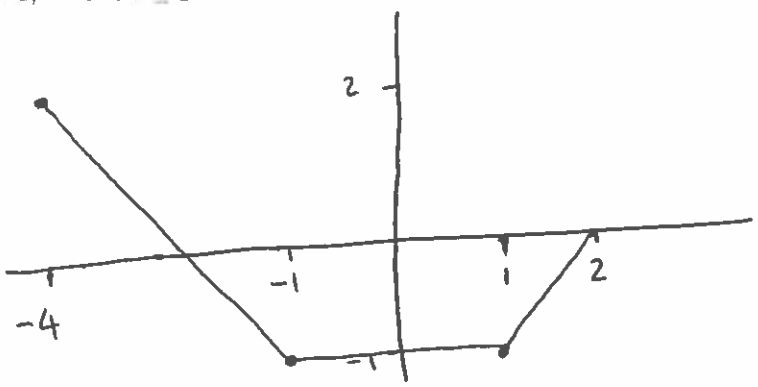
(d) $(g \circ g)(x)$

$g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}}}} = \sqrt{\sqrt{x+2}} = \sqrt[4]{x+2}$

Answer: $(g \circ g)(x) = \sqrt[4]{x+2}$

6. Sketch the function

(a) $f(x) = \begin{cases} -x-2, & -4 \le x \le -1 \\ -1, & -1 < x \le 1 \\ x-2, & 1 < x \le 2 \end{cases}$



7. Describe, in words, how each graph is obtained from the graph of $y = f(x)$.

(a) $y = f(x - 5)$

Answer: Shift right 5 units

(b) $y = f(4x)$

s.f = scale factor

Answer: Stretch s.f 1/4 in x direction

(c) $y = f(-3x)$

Answer: Reflect in y-axis + stretch s.f 1/3 in x direction

(d) $y = f(2x + 1)$

Note, order matters here.

Answer: Stretch s.f 1/2 in x direction then shift left 1 unit

(e) $y = f\left(\frac{x}{3}\right) - 4$

Answer: stretch s.f 3 in x direction then shift down 4

(f) $y = -3f(x) + \frac{1}{4}$

Answer: Stretch s.f 3 in y direction and reflect in x-axis. Then shift up 1/4 units

Cont.

8. Express the radius of a sphere as a function of the sphere's surface area. Then express the surface area as a function of volume.

$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

Solve for $r$ in terms of $V$.

$= 4\pi\left(\frac{9V^2}{16\pi^2}\right)^{1/3} = \left(\frac{4^3\pi^3 9V^2}{16\pi^2}\right)^{1/3}$

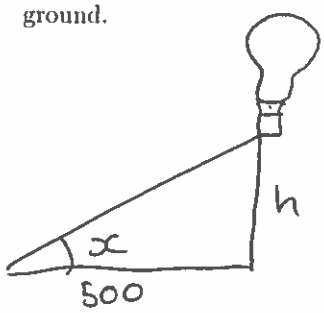
$= (4\pi 9V^2)$

$\Rightarrow r^3 = \frac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

$= \left(\frac{3V}{4\pi}\right)^{1/3}$

Answer: $S = (36\pi V^2)^{1/3}$

9. A hot air balloon rising straight up from a level field is tracked by a range finder located 500m from the point of liftoff. Express the balloon's height as a function of the angle the line from the range finder to the balloon makes with the ground.




← Badly drawn balloon.

$\frac{h}{500} = \frac{opp}{adj} = \tan(x)$

$\Rightarrow h = 500\tan(x)$

Answer: $h = 500\tan(x)$

10. If Harry invests £1500 in a retirement account and earns 8% compounded annually, how long will it take this single payment to grow to £5000? If Ron invests £2000 and earns 5%, who will reach £7000 first.

$1500(1{\cdot}08)^x = 5000 \Rightarrow x = \frac{10}{3\ln(1{\cdot}08)}$ years

$1500(1{\cdot}08)^x = 7000 \Rightarrow x = \frac{14}{3\ln(1{\cdot}08)}$ $2000(1{\cdot}05)^x = 7000 \Rightarrow x = \frac{14}{4\ln(1{\cdot}05)}$

$\frac{14}{3\ln(1{\cdot}08)} - \frac{14}{4\ln(1{\cdot}05)} = \frac{7(4\ln(1{\cdot}05) - 3\ln(1{\cdot}08))}{6\ln(1{\cdot}08)\ln(1{\cdot}05)} = \frac{7}{6\ln(1{\cdot}08)\ln(1{\cdot}05)}\ln\left(\frac{1{\cdot}05^4}{1{\cdot}08^3}\right)$

$\frac{1{\cdot}05^4}{1{\cdot}08^3} < 1 \Rightarrow \ln\left(\frac{1{\cdot}05^4}{1{\cdot}08^3}\right) < 0 \Rightarrow$ Ron.

Answer:

I've decided q.10 is too hard for an exam.

The End.